Competition and Unconscionability

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Abstract

Conventional doctrine holds that courts should be more willing to refuse to enforce suspect clauses in contracts on the grounds of unconscionablity when either 1) The seller or proponent of the clause had monopoly power or 2) The buyer was not given a choice of contracts or an opportunity to bargain. This paper suggests that this doctrine is misguided on both points. I argue that a major rationale for refusing to enforce contracts is the presence of naive customers who are vulnerable to exploitation. I show that if there are such naive customers, when a seller with substantial market power is offering only one contract to all customers, fear of alienating sophisticated customers disciplines the seller and discourages it from using inefficient contracts in order to exploit the naive. On the other hand, in a more competitive industry, sellers will obtain most of their profits from naive customers and may actually be losing money on the sophisticated customers. Thus they will not be concerned with losing sophisticated customers and will be willing to use inefficient and exploitative contracts to extract from the naive. This paper presents a formal model which shows that under broad conditions exploitation of the naive is increasing in the number of firms in a market. Sophisticated customers will generally be made better off by competition, either they will be able to avoid most of the costs of exploitation and be cross-subsidized by the naive, or they will purchase from firms which focus on sophisticated customers and offer efficient contracts, while the naive purchase from firms that advertise low prices but offer maximally exploitative contracts. The effects of competition on the naive customers is ambiguous. Competition results in lower prices for these customers, but more inefficient exploitation.

1 Introduction

Conventional doctrine on unconscionability holds that courts have more justification in rejecting contract clauses in the "absence of meaningful choice on the part of one of the parties."[1] In *Henningsen v. Bloomfield Motors, Inc.*, the New Jersey Supreme Court refused to sustain a waiver of warranty rights covering automobiles on the grounds that virtually all automakers insisted upon such a waiver, arguing that "[t]here is no competition among the car makers in the area of the express warranty."[2] The intuition here is that a monopolist or a party with considerable bargaining power can insist on terms that disadvantage the purchaser. However this argument overlooks the fact that in most cases, a party with considerable bargaining power can more profitably use that power to increase the price of the good. If the contract term is inefficient, then including the term decreases the buyer's willingness to pay more than it benefits the seller. Thus the seller is better off selling the good at a higher price without the inefficient clause. This paper is by no means the first to argue that monopolists have no more incentive than competitive firms to include terms that are unfavorable to the buyer. It is by now widely understood that when dealing with customers who properly interpret the terms of the contract, a monopolist will exploit market power primarily through price and does not generally have an incentive to insert terms that are inefficient in the contract.[3]

[1] *Williams v. Walker-Thomas Furniture*, 121 U.S. App. D.C. 315; 350 F.2d 445,449

[2] *Henningsen v. Bloomfield Motors, Inc.*,32 N.J. 358,391, 161 A.2d 69,87 (N.J 1960)

[3] See for example, Alan Schwartz, *A Reexamination of Nonsubstantive Unconscionability*, 63 Virginia Law R. 1053 (1977), which makes this point and nicely argues that to the extent that a form contract contains terms that are inefficient for a particular customer, they are likely to be efficient when taking into the account the costs of individualizing contracts, and that a monopolist has no less incentive to individualize contracts than competitive firms.

To the extent that a monopolist does have an increased incentive to introduce inefficient contract terms when dealing with rational consumers, it is most likely an attempt at 'metering' or some other way of imposing price discrimination. (See Richard Schmalansee *Monopolistic Two-Part Pricing Arrangements,* 8 Bell Journal of Economics 445 (1981), see also Walter Oi, *A Disneyland Dilemma: Two-Part Tariffs for a Mickey Mouse Monopoly*, 85 Quarterly J. of Econ 77 (1971)). When consumers differ in their willingness to pay for a good, monopolists might impose inefficient terms on those who value the good less to extract more surplus from those who value the good more. One common example of privately inefficient terms is the imposition of advance purchase requirements and large change fees on cheaper tickets by airlines. By imposing these inefficient requirements and fees on low priced tickets, airlines make low price tickets less attractive to people who value flexibility highly, and encourage them to purchase full fare tickets. (On the other hand see James D. Dana, Jr., *Advance-Purchase Discounts and Price Discrimination in Competitive Markets,* 106 Journal of Political Economy 395, (1998), which argues that this can occur with competitive firms, and is basically efficient)). However, economic theory suggests that the welfare effects of price discrimination are ambiguous, so this is not a strong argument for extra scrutiny of the contracts of monopolists.

When both parties to a contract are fully informed and can correctly anticipate the costs and benefits of any term, neither party ordinarily has an incentive to insert inefficient terms into a contract. Nonetheless, courts will occasionally strike down terms they viewed as oppressive in a contract even when there is no evidence that the disadvantaged party was unaware of the term. More recent research into behavioral economics provides a more powerful justification for refusing to enforce specific contract terms, namely, suppliers might design contracts to take advantage of behavioral biases or other cognitive limitations of consumers. If a buyer incorrectly predicts the effects of a contract term favorable to the seller, the seller may have an incentive to introduce the term, even when it is inefficient. Because the buyers do not take into account the full cost of the term when choosing to enter the contract, their willingess to pay is not decreased by the entire cost of the term to them. Authors such as Bar-Gill[4] and DellaVigna and Malmendier[5], have provided evidence of marketers taking advantage of their customers' cognitive limitations by using contracts of a form that would be unlikely to be seen without some behavioral explanation.

The industries which DellaVigna and Malmendier and Bar-Gill write about, health clubs and credit cards, respectively, are quite competitive. Another example of apparently inefficient contract terms in competitive industries is the use of late fees that were substantially greater than rental fees in video stores.[6] The fact that these contracts appear in industries which are quite competitive suggests that competition is not sufficient to protect naive consumers from exploitative contracts. Other authors, notably Laibson and Gabaix,[7] have shown that competition will not always protect unsophisticated customers from attempts to exploit their ignorance or misapprehension of the import of contract terms.

The primary contribution of this paper is to take this rationale for firms to introduce inefficient terms into their contracts, and to show that there are plausible conditions where the presence of competition is a reason for *more* scrutiny of contract terms rather than less. In this paper, I introduce buyers with a type of cognitive failure that is likely to lead sellers to offer contracts with terms that are inefficient

[4]See Oren Bar Gill *Seduction by Plastic* 98 NORTHWESTERN LAW REVIEW 1373 (2004)

[5]See Stefano DellaVigna and Ulrike Malmendier, *Contract Design and Self-Control: Theory and Evidence*, 119 QUARTERLY J. OF ECON.353 (2004)

[6]Prior to clas action litigation in 2001, in 1999 Blockbuster's revenue from 'Extended Viewing Fees' amounted to % 18.4 of rental revenue. (Blockbuster form 10-K405 filed 3/29/2001, p37). "Until February 2000, Blockbuster customers who kept films beyond the specified rental period were assessed a late penalty that was equal each day to the initial rental fee. If the rental fee was $2.99, for example, a customer who returned a tape five days late would be charged a penalty of $14.95." – Geraldine Fabrikant,*Blockbuster Settles Suits On Late Fees*,NYT Published: June 6, 2001. Bockbuster faced about two dozen such suits.

[7]Xavier Gabaix and David Laibson *Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets,* 121 QUARTERLY J. OF ECON. 505 (2006)

and consider the question of whether sellers have less incentive to include inefficient exploitative terms when they are selling goods in competitive markets. I find the opposite. Competition will fail to provide any protection to naive consumers from excessive exploitation under some plausible conditions where monopoly will offer at least some protection. Specifically, I show that firms have more incentive to use grossly inefficient contracts when there is more effective competition in the market or where consumers are given a choice of contracts.

The intuition here is that when firms require customers to agree to exploitative contract terms they lose sophisticated customers, those who understand the exploitative term. A firm that is realizing substantial profits from all of its customers will consider the effect the exploitative term has on the demand from these sophisticated customers. At the same time, when sophisticated customers have some ability to avoid or mitigate the consequences of the inefficient term, firms will realize lower profits from their sophisticated customers than from their naive customers. As the industry becomes more competitive, the profit from the average customer decreases, approaching zero as the industry becomes perfectly competitive. Since the firms are making more from their naive customers, eventually they are making little or nothing from their sophisticated customers, and have diminished incentive not to alienate the sophisticated customers by inserting more exploitative inefficient terms into the contract. On the other hand, for an oligopolist who is able to profit on all of its customers, the profits that the firm realizes from its sophisticated customers acts as a disciplining device on the firm's proclivity to exploit.

Laibson and Gabaix's recent work on shrouded attributes considers a similar model, but they focus on a different question: whether competition provides a firm an incentive to educate the public about its competitor's exploitative behavior. Rather than focusing on the incentive of firms to educate naive consumers, this paper takes the cognitive failures of the consumers as exogenous and focuses on how the incentives of firms to exploit their consumers' naivete is affected by competition.

In the model I present, firms would like to offer different contracts to different types of customer. Specifically, they would like to offer sophisticated customers efficient contracts while exploiting the naive as much as possible. A key assumption of the model in section 2 of this paper is that firms offer only one contract to their customers. Empirically this is true of many, yet not all industries. For example, cellular telephone services will explicitly offer customers a menu of contracts, whereas car rental firms and hotels will typically only offer only one contract to walk-up customers. The administrative costs of offering a choice of different contracts and explaining them to potential customers might be enough to dissuade firms from offering a menu of contracts. Another reason why firms may not offer menus of contracts is that it might be impossible to do this without pointing out the exploitative nature of the contract aimed at the naive. For example a video rental firm would not be able to offer a contract with reasonable late fees at a high rental price without pointing out that they charge unreasonable late fees in the standard contract. Similarly reputation effects may prevent a firm from offering different contracts; if customers hear that a firm is offering some exploitative contracts, they might assume that all of the contracts offered by the firm are exploitative, and it might be costly to convince them otherwise.

If firms can freely design contracts aimed at individual customers, firms would view the two types of consumers as two different markets. Thus they could offer efficient contracts that are attractive to the sophisticated and maximally exploitative contracts at lower prices to tempt the naive. The fact that a firm is offering a choice of contracts that vary in the degree of exploitation indicates that the presence of sophisticated consumers is unlikely to provide much protection to naive consumers, regardless of whether the industry is a monopoly or is competitive. Thus the consumers who are most in need of protection might be those who face a choice of contract terms.

However when there are fixed costs to discriminating in this way, competition may offer some protection to naive customers. As shown by Alan Schwartz, the returns to designing a contract to exploit naive customers in a monopolistic industry may be sufficient to induce the monopolist to design such a contract, whereas in a competitive industry the producer might not see a large enough market to justify the fixed costs of the contract design.[8]

Even when sophisticated customers do not have any ability to lessen the impact of the exploitative term, firms with market power will have less incentive to use grossly exploitative contracts. Specifically, when firms have significant market power, each firm will serve a cross section of the market, thus they will balance the benefit they obtain from exploiting the naive with the cost of imposing an inefficiency on the sophisticated. However as more firms enter a market, the profits available from offering a standard contract to a cross section of the market decreases, and it becomes worthwhile for some firms to become 'niche' producers and focus on the naive market. These firms will offer deals that are 'too good to be true'; they will offer the good at a low price but since they are focusing on the naive market they will offer as many exploitative terms as possible.

Competition tends to be good for the sophisticated. Sophisticated customers will tend to pay lower prices under competition, and as the market becomes perfectly competitive they will either be offered contracts without any exploitative terms, or will be cross-subsidized by the naive customers to the extent that they are better off than they would be under an efficient contract offered at cost. On the other hand, the effects of competition on the welfare of the naive is ambiguous. Under an oligopoly, the profits the firms make from the sophisticated customers constrains their

[8]Alan Schwartz, *How Much Irrationality Does the Market Permit?*, 37 J. of Legal Stud. (2008)

incentives to exploit the naive. When there is more competition, naive customers will purchase maximally exploitative contracts from firms that do not make money off the sophisticated. When the inefficiency from exploitation outweighs the lower mark-ups from competition, naive customers are worse off. On the other hand if there are few sophisticated customers, even monopolists will have little reasons not to exploit the naive, and in a competitive market, the naive will at least pay a lower price. Likewise if the degree of possible exploitation is small, naive customers may be better off being fully exploited but paying a low price compared to paying a monopoly price with a more efficient contract.

This paper is not intended to be an argument for legal protection of oligopolies in general, rather it is intended to shed light on the circumstances where the terms of a contract should be more closely scrutinized for inefficient exploitation. Taking the view that courts should question the enforceability of contract terms only when they are inefficient and would not have been agreed to by fully informed parties, this paper suggests that we are more likely to see such terms in a competitive market. In other words, and contrary to the traditional doctrine on contracts of adhesion, in cases where exploitative contract terms are alleged a competitive market structure should be viewed as a strike against rather than a point for the seller.

2 Example

To illustrate the intuition behind our major result, we compare two hypothetical firms, both about the same size. The first operates in a small town and is a monopolist, the second operates in a larger city, and thus faces more competition (but not perfect competition). Both firms serve two types of customers: *naifs* who only look at the upfront price, and *sophisticates* who correctly anticipate all the costs of the contract.

The monopolist charges $5.70 upfront for a service that costs $5.00 to provide, but also insists upon an inefficient exploitative clause in the contract. This clause ends up costing each naif $2.00, but since the naifs only look at the upfront price, they do not anticipate these costs and the term does not impact their demand. The clause nets the firm an additional $1.60 per naive customer, so the firm's net profit from each naif is $2.30. Sophisticated customers are able to anticipate and some times avoid the effect of this clause, so it ends up costing them less. We assume, the clause costs them $1.50 on average, and the firm only nets $1.20 per sophisticated customer from the clause's effects. The firm's profit from each of its sophisticated customers is thus only $1.90.

Suppose that the monopolist serves 300 sophisticates and 100 naifs, and would lose 4 customers (3 sophisticates and 1 naif) in response to a $.02 price increase. In this case the monopolist could not increase profits by increasing or decreasing the price. Now suppose the monopolist's lawyer proposes an extra exploitation subclause that would increase net revenue from naifs by $.04 per customer, but would cost each naive customer $.08 This subclause would cost the sophisticates $.06 and would bring in .03 per sophisticate. Since the naifs don't anticipate being exploited, the firm wouldn't lose any naive customers as a of inserting the subclause, but the sophisticated customers would see the subclause as a price increase of $.06, so the firm would lose 9 sophisticates. Thus the firm would increase its take from naifs by a total of $4.00, and it would increase its take from its 291 remaining sophisticates by $8.73, but it would lose $17.10 in profits from the 9 lost sophisticated customers, which would outweigh the gains from the remaining customers. The firm could mitigate these losses by reducing prices slightly,[9] but it would still be unprofitable to add this extra exploitation term.

Now we consider the competitive firm, which has the same costs, but faces competition, which makes its customers much more sensitive to price. As a result it charges a lower base price of $3.90. With the same first exploitation clause, it brings in another $1.60 per naif, and $1.20 per sophisticate, so its net profit is $.50 per naif and $.10 per sophisticate. Suppose the competitive firm serves 300 sophisticated customers, and 100 naive customers. These prices are profit maximizing if the competitive firm's customers are ten times more sensitive to price than the monopolist's so the competitive firm would lose 30 sophisticates, and 10 naifs in response to a $.02 price increase.

Now we ask whether the competitive firm would want to add the extra exploitation subclause (We assume that it costs naifs and sophisticates $.08 and $.06, respectively and brings in $.04 and $.03, as before). If the firm added the subclause and held price constant, it would lose 90 sophisticates, and lose $9.00 of profits from them, but it would gain $4.00 from the naifs and $6.30 from the remaining sophisticates. Because inserting the subclause reflects an effective increase in price, it could increase its gains further by lowering the price a few cents to compensate.[10]

In this example, the monopolist does not find it profitable to increase its exploitation because doing so will drive away too many sophisticated customers. Although the monopolist could lower prices to compensate, doing so would negate all the gains from increased exploitation. On the other hand, the firm in the more competitive environment does find it profitable to insist on the term; because the competitive firm is making so little on the sophisticated customers, its losses from alienating the sophisticates is relatively small. Competition has more effect proportionately on the profits from the sophisticates (which are decreased in this example by 94.5%), than

[9]By dropping its base price to $5.66 it would recover $.30 in lost profit

[10]If it lowered the price by $.04 in conjunction with the new term, it would increase by another $3.00

on the profits from the naifs (which are decreased by only about 78% in this case). By making sophisticated customers less important in the market, competition increases the likelihood that firms will find it profitable to focus on naive customers by offering exploitative contracts. Although we considered the firms willingness to adopt a small increase in exploitation, the results would be the same if we had considered a large change in exploitation. The next section presents a formal model

3 Model

We consider a mass 1 of consumers (indexed by $k \in [0,1]$) who are considering purchasing a good from at most one of J producers. Each firm $i \in J$ offers a good for sale that has a production cost of 0 with a contract which consist of two terms, a price p_i and an exploitation term e_i. Unlike Schwartz,[11] we assume that the firm is constrained to offer the same price and the same contract to all consumers. As discussed in the introduction, if firms are able to costlessly offer different terms to different customers, they will exploit the naive regardless of market structure. We assume that the cost of producing the good is 0 and we allow for negative prices.[12] The function $f(e)$ describes the firm's returns from exploitation, so the revenue the firm realizes from a sale to a customer who does nothing to avoid the exploitation term is $p_i + f(e_i)$.

We use a random utility model where consumers vary in their preferences for the goods offered by the various firms. The random, firm specific, taste shock model we use is suitable for the monopoly model, and leads to tractable results for differentiated Bertrand competition when we introduce multiple firms in the next section. The utility each consumer k receives from consuming from firm i is $v + \epsilon_{ik}$, where ϵ_{ik} is an idiosyncratic taste shock drawn independently for each i and k from the unit uniform distribution. Thus $\epsilon_{ik} \sim U[0,1]$, so for any $z \in [0,1]$, the likelihood that $\epsilon_{ik} \leq z$ is equal to z. The uniform distribution for the taste shock is chosen for convenience; in subsection 4.1 we show that our basic results are robust to changes in the taste distribution.

We explain the presence of inefficient exploitation by assuming that consumers differ in the ability to understand and respond to contracts. Specifically, we assume that a fraction λ of the consumers are 'naive', and they ignore the potential for exploitation when purchasing the good and are also unable to do anything to reduce the exploitation after they have purchased the good. The cost to a customer who does nothing to reduce the exploitation is $p_i + e_i$, however, naive customers only anticipate

[11] Supra, note 10

[12] Our results do not require that the actual price charged to consumers is negative. Given our assumption that production cost is 0, the reader can interpret p as a margin rather than a price.

paying p_i.

The remaining $1-\lambda$ customers are 'sophisticated'. These consumers correctly anticipate the effects of all clauses in the contract. Because they understand the contract, we assume they also have some ability to mitigate the effects of the exploitative clauses. A sophisticated customer purchasing from i avoids a share $(1-\alpha)$ of the exploitation, so the sophisticated customer's total costs from contract i will be $p_i + \alpha e_i$, and this will also be their anticipated costs. When the sophisticated customer avoids exploitation, it naturally reduces the amount of revenue the firm realizes from exploitation. Because the sophisticated consumer is able to avoid $1-\alpha$ of the exploitation the firm only receives $\alpha f(e_i)$ from exploiting the sophisticated customer. Thus the sophisticated customers differ from the naive in two important ways: Firstly, they correctly understand the contract and predict the costs associated with it. Secondly, they are able to mitigate the exploitation by modifying their behavior to avoid the exploitative clauses. In subsection 4.2, I analyze the model where sophisticated customers are unable to avoid exploitation, so they differ from the naive only in their ability to predict the consequences of the contract.

We assume that f is continuous and differentiable and that $f'(0) = 1$ and $f''(e) < 0 \forall e < \bar{e}$. This implies that for any $e > 0$, $e - f(e) > 0$ so the cost to the customer from exploitation is always greater than the revenue the firm receives, and any exploitation is socially inefficient. We also assume that there is some $\bar{e} < \infty$ such that $f'(\bar{e}) = 0$ implying that there is an upper bound to the amount that a firm will exploit even a naive customer. One might interpret this upper bound as an acknowledgement that there are probably some costs to exploiting even naive customers (perhaps bad publicity) and that there are limits to what a firm would be able to extract.

For the sake of compactness we will define $\beta{=}\lambda + (1-\lambda)\alpha$, so that β is the 'average' impact of exploitativeness on the consumers,. Thus, if a seller's consumers are representative in term of sophistication, her per customer returns from exploitation are $\beta f(e_i)$.

3.1 Competition

We now turn our focus towards characterizing an equilibrium with multiple firms, with the number of firms denoted by $J > 1$. As noted above, we are assuming that competition is not perfect, rather consumers have idiosyncratic preferences for the good offered by each firm, so as long as there are a finite number of firms, each firm has some market power. However, the greater number of firms, the more likely a customer sees a close substitute to the product offered by any particular firm, and the less market power each firm has. For the purpose of simplicity we will confine our discussion to the cases where the minimum value of the good to consumers(v), or the number of firms (J) is high enough so that in equilibrium all consumers wish to purchase from at least one firm.[13]

If p_i and p_j are the prices being charged by firms i and j respectively, and ϵ_{ik} is consumer k's idiosyncratic preference for i, given that $e_{jk} \sim U[0,1]$, the likelihood that a sophisticated k prefers to purchase from i rather than j is

$$\min(1, \max(0, \epsilon_{ik} + p_j + \alpha e_j - p_i - \alpha e_i)) \tag{1}$$

The likelihood that a naive k prefers to purchase from i rather than j is:

$$\min(1, \max(0, \epsilon_{ik} + p_j - p_i)) \tag{2}$$

Let us define x_i^n as sales to naifs and x_i^s as sales to sophisticates. Since ϵ_{ik} is uniform, the total sales by firm i are given by:

$$x_i = x_i^n + x_i^s = \lambda \int_0^1 \prod_{j \neq i \in J} \min(1, \max(0, \zeta + p_j - p_i)) d\zeta +$$
$$(1-\lambda) \int_0^1 \prod_{j \neq i \in J} \min(1, \max(0, \zeta + p_j + \alpha e_j - p_i - \alpha e_i)) d\zeta \tag{3}$$

If all firms other than i are acting symmetrically, and offering a contract with terms p_j and e_j the naifs' and the sophisticates' demand for firm i's are given by:

$$x_i^n = \lambda \int_{\max(0, p_i - p_j)}^{\min(1, 1 - p_j + p_i)} (\zeta + p_j - p_i)^{J-1} d\zeta + \max(0, p_j - p_i) \tag{4}$$

$$x_i^s = (1-\lambda) \int_{\max(0, p_i - p_j + \alpha e_i - \alpha e_j)}^{\min(1, 1 - p_j + p_i - \alpha e_i + \alpha e_j)} (\zeta + p_j + \alpha e_j - p_i - \alpha e_i)^{J-1} d\zeta + \max(0, p_j - p_i + \alpha e_j - \alpha e_i) \tag{5}$$

Note that in a symmetric equilibrium with J competitors, each competitor's market share of both naive and sophisiticated consumers will be $\frac{1}{J}$. Furthermore if we differentiate (5) where $p_i = p_j$ and $e_i = e_j$, we have $\frac{\partial x_i^s}{\partial p_i} = 1-\lambda$, $\frac{\partial x_i^n}{\partial p_i} = \lambda$, $\frac{\partial x_i^s}{\partial e_i} = (1-\lambda)\alpha$, and $\frac{\partial x_i^n}{\partial e_i} = 0$. Profits are given by $\pi_i = x_i^s(p + \alpha f(e_i)) + x_i^n(p + f(e_i))$. We note that if naifs are a share λ of the firm's customers, the average profit per customer will be $p_i + \lambda f(e_i) + (1-\lambda)\alpha f(e_i) = p + \beta f(e_i)$.

Our first order conditions for price and exploitation are as follows:

$$\frac{\partial \pi_i}{\partial p_i} = 0 \Leftrightarrow \frac{1}{J} = p_i + \beta f(e_i) \tag{6}$$

$$\frac{\partial \pi_i}{\partial e_i} = 0 \Leftrightarrow \frac{\beta f'(e_i)}{J} = \alpha(1-\lambda)(p_i + \alpha f(e_i)) \tag{7}$$

[13] A sufficient condition for all consumers to purchase is that $\min[v, v - \alpha \bar{e} + \beta f(\bar{e})] > \frac{1}{J}$.

By (6) this is $\frac{1}{J}$, so profit will be $\frac{1}{J^2}$. This is consistent with the standard models of Hotelling competition on a circle, where profit decreases with the square of the number of competitors. Returning our focus to the returns to exploitation and substituting in from (6) we can write the first order condition as

$$\frac{\beta f'(e_i)}{J} = \alpha(1-\lambda)(\frac{1}{J} - (\beta - \alpha)f(e_i)) \tag{8}$$

or

$$\beta f'(e_i)(p_i + \beta f(e_i)) = (1-\lambda)\alpha(p_i + \alpha f(e_i)) \tag{9}$$

Equilibrium competitive exploitation e_i is implicitly given by:

$$f'(e_i) = \frac{\alpha(1-\lambda)(p + \alpha f(e_i))}{\beta(p + \beta f(e_i))} \tag{10}$$

Examining (8), we note that increasing J decreases the benefit of exploitation represented by the left side of (8) proportionally less than it decreases the cost, represented by the right side. The intuition behind this result is that as competition increases, the ratio of profit a firm gets by serving naive customers to the profit it gets from serving sophisticated customers increases. As a result, firm is more willing to risk losing sophisticated customers by increasing the exploitativeness of its contracts. Furthermore, as $(\beta - \alpha)f(e_i)$ approaches $\frac{1}{J}$, the profit the firms are getting from sophisticated customers $(p + \alpha f(e_i))$ approaches zero. Thus the sophisticated customers exert no disciplining influence, and firms will choose contracts of maximum exploitativeness.

Not surprisingly, increasing λ increases β and thus increases the left side and decreases the right side; as the proportion of naive customers increases, exploitation becomes more attractive. Increasing α also increases β but it increases the ratio$\frac{\alpha}{\beta}$, so an increase in α causes the lost business from exploitation (right side of (8)), to increase relative to the increased revenue per customer on the left side. Decreasing the ability of the sophisticated to avoid exploitation does increase the revenue a firm receives from exploitation from a particular sale, but it makes exploitation less attractive, because it causes more sophisticated customers to avoid exploitative contracts.

3.2 How does competition affect exploitation?

Although the above comparison is suggestive that equilibrium exploitation is increasing in competition, it falls somewhat short of a proof. We note that equation (8) is a first order condition which represents a necessary, but not sufficient condition for equilibrium. The difficulty is that without further assumptions, there could be several values of e which satisfy (8) for a given J, each representing a local maximum, where firms would have no incentive to engage in small deviations. However, we are able to obtain our major result by ruling out the existence of multiple symmetric equilibria for any given set of parameter values.

Proposition 1 *Suppose $\alpha < 1$, equilibrium exploitation (e_i) is (weakly) increasing in the number of competitors. This inequality is strong as long as $e_i < \bar{e}$.*

Proof: See Appendix

The proof proceeds by showing that although there may be several local maxima (candidate equilibria), at most one of these candidates will represent a symmetric equilibrium. Furthermore as J increases only lower priced higher exploitation candidates can be equilibria, so as J increases, exploitation increases either because exploitation is increasing at each local maximum, and the same local maximum represents an equilibrium, or because the new equilibrium is at a higher exploitation local maximum.

3.3 Bifurcated equilibrium

Even when there is a unique solution to (8), this guarantees only that the firms are locally maximizing their profits. It is possible that a firm might be able to increase its profits with a large deviation. We can think of a large deviation as pursuing a niche strategy. If the candidate symmetric equilibrium is a local maximum, initially a deviation towards a lower priced but more exploitative contract begins to decrease profits because the marginal returns to exploitation are decreasing and the marginal inefficiency of exploitation is increasing. However as the deviant firm increases exploitation and drops price, the firm gains naive customers and loses sophisticated customers. As the firm serves fewer sophisticated customers, the marginal loss of business from increases in exploitation shrinks, and further increases in exploitation can increase profits. Similarly, if all other firms are offering very exploitative contracts, it might be profitable for a firm to make a large deviation in the other direction, focusing on sophisticated customers by offering a contract with a relatively high price, but little or no exploitation.

However, even as J becomes large there will not always be a profitable large deviation. If sophisticates are able to avoid a fraction of the exploitation, they receive a cross subsidy from the naive customers who are being fully exploited. If this cross subsidy is greater than the efficiency loss from the inefficient exploitation, the sophisticates prefer a firm that offers the exploitative contract at the (below cost) competitive price to a firm that offers the efficient contract at cost. Formally, as

$J \to \infty$ as long as $\alpha < 1$ the level of exploitation that solves (8) reaches $\bar{e}$. Now suppose the returns to exploitation are great enough and there are enough naive customers, so that $\frac{f(\bar{e})}{\bar{e}} > \frac{\alpha}{\beta}$. Since profit per customer goes to zero as $J \to \infty$, in the symmetric equilibrium, $p \to -\beta f(\bar{e})$, so if $\frac{f(\bar{e})}{\bar{e}} > \frac{\alpha}{\beta}$, a deviant could not attract a significant number of sophisticated customers by offering a contract with with $e_i = 0$ and $p_i = \epsilon > 0$. What is occurring in that case is that the naive customers are essentially subsidizing the sophisticated customers and it is not possible to make a profit by serving only sophisticated customers. On the other hand if $\frac{f(\bar{e})}{\bar{e}} < \frac{\alpha}{\beta}$ then the profits the firms get from exploiting the naive is not sufficient to compensate all of the sophisticated for the inefficiencies from the exploitation they endure. A firm could profitably serve at least some of the sophisticated by offering a no exploitation contract at a positive price. The condition here is analogous to the condition in equation (13) of Gabaix and Laibson, and is simply a comparison of the cross subsidy-from naive to sophisticated customers with the deadweight loss.

Note that as the firms serves a higher fraction of naifs in the competitive equilibrium, the cross subsidy increases. Although there may be few enough naive customers so that the cross subsidy is not enough to entice sophisticates to take the inefficient contract when a firm is serving a cross section of consumers, the cross subsidy might be large enough to draw a sophisticate to a firm that is only serving naifs. In equilibrium, as deviating firms siphon off some of the sophisticated (on whom the exploitative firms are losing money), the exploitative firms lower price, and it may become impossible for the deviating firms to siphon off more sophisticates. Formally, if $\alpha < \frac{f(\bar{e})}{\bar{e}} < \frac{\alpha}{\beta}$ then there will be enough of a cross subsidy to convince at least some sophisticated customers to take the exploitative contract. In this case sophisticated customers will be split between firms that offer an upfront price below cost but high exploitation, and firms that offer efficient contracts priced just above cost. All naive customers will choose firms offering the low price / high exploitation contract.

On the other hand if $f(\bar{e}) < \alpha\bar{e}$, even when a firm is serving only naifs and making zero profit, the cross subsidy is not enough to make up for the efficiency loss from exploitation, and it will always be possible to attract sophisticates by offering an efficient contract at cost. In this case we will have complete bifurcation. Thus as the number of firms gets large, the market equilibrium will be one of three classes, described by the following proposition.

Proposition 2 *As J becomes large, if (a) $\alpha\bar{e} < \beta f(\bar{e})$ there is a symmetric equilibrium where each firm offers the naive contract ($p \approx -\beta f(\bar{e}) + \frac{1}{J}, e = \bar{e}$). If $\beta f(\bar{e}) < \alpha\bar{e}$ there will be a bifurcated equilibrium. If (b) $\beta f(\bar{e}) < \alpha\bar{e} < f(\bar{e})$ there is an equilibrium where all naive consumers and a fraction $\mu \to \hat{\mu} = \frac{\lambda}{1-\lambda}\frac{f(\bar{e}-\alpha\bar{e})}{\alpha(\bar{e}-f(\bar{e}))}$ of sophisticated customers choose the 'naive' contract with $p \approx -\alpha\bar{e}$ and $e = \bar{e}$, and the remaining sophisticated consumers choose the 'sophisticated' contract with $p \approx 0$ and $e = 0$. In this* *equilibrium the number of firms offering the sophisticated and naive contract respectively (J_S and J_N) is approximated by $J_S^3 \approx \frac{(1-\mu)^2 J_N^2}{\mu\alpha(\bar{e}-f(\bar{e}))}$. If (c) $f(\bar{e}) < \alpha\bar{e}$ then there is a bifurcated equilibrium where all sophisticated types choose the sophisticated contract and all naive types choose the 'naive' contract ($p \approx -f(\bar{e}), e = \bar{e}$) and $\frac{J_N^2}{J_S^2} = \frac{\lambda}{1-\lambda}$*

Proof: See Appendix

Note $\hat{\mu}$ is defined so that if the naive firms serve a fraction $\hat{\mu}$ of all sophisticated customers, and naive firms charge a price $-\alpha\bar{e}$ they would make zero profit. Interestingly part(b) of the proposition implies that if $\beta f(\bar{e}) < \alpha\bar{e} < f(\bar{e})$ as J becomes large, the fraction of firms that are offering the 'sophisticated' contract approaches zero, although the fraction of customers served by these firms remains roughly constant. The profit per customer for firms offering the sophisticated contract approaches zero faster than the profit per customer for firms offering the naive contract. The intuition for this is subtle and is similar to the intuition presented by Lawrence Ausubel regarding the limited effect of competition among credit card issuers.[14]. As long as sophisticated customers are being served by firms offering both types of contract, their elasticity to price will be greater than the price elasticity of naive customers, who are being served only by firms offering naive contracts. Because the sophisticated customers are slightly more elastic (they have more choices) they represent a disproportionate share of the marginal customers that a firm would gain in response to a price decrease. Since the firm offering the naive contract makes money on naive customers but loses money on sophisticated customers, the fact that sophisticated customers are more elastic implies that when the firm is breaking even on the marginal customer, they are making money on the average customer. Thus the profits per average customers drops more slowly for firms aiming at the naive, in order to keep firms indifferent between the two strategies, a growing proportion of firms must choose to serve the naive.

Note that in all three cases, as the number of firms grows large, all naive customers are being maximally exploited. In cases (b) and (c), in fact, they are hurt by the presence of sophisticated customers rather than helped. Because the firms extract less from the sophisticated customers, they must charge a higher upfront price in a competitive equilibrium, so the although the presence of sophisticated customers does not decrease the exploitation the naifs face, it does increase the price they pay.

[14]See Lawrence Ausubel, *The Failure of Competition in the Credit Card Market*, 81 Am. L. & Econ. Rev 50, (1991)

4 Robustness

Our results so far have depended upon a number of special assumptions. We have assumed that there are only two types of customer, and that the naifs are totally naive, that is they do not anticipate or prevent exploitation at all. We have also assumed that the sophisticated are able to avoid at least some exploitation Furthermore, we have assumed the idiosyncratic taste parameter had a very specific distribution. In this section, we look at the consequences of relaxing these assumptions, and show that for the most part, our major results are robust.

4.1 Generalizing the model of competition

Because it can be difficult to achieve simple results for equilibria in general models where firms compete for consumers who vary in several dimension, our results so far have been developed using a very specific model of competition. To wit, we assume that individuals' idiosyncratic taste parameters followed a particularly convenient distribution. Since it is unlikely that the distribution of individual tastes corresponds exactly to our assumption it would be troubling if our results were very sensitive to this assumption. In this subsection we show this is not the case. In fact, our first order condition for exploitation (10) does not depend at all upon the precise shape of the distribution. As long as idiosyncratic taste parameters are independent and identically distributed, we show that in any symmetrical equilibrium with J firms, if all customers purchase the good, exploitation will be decreasing in price. As long as price is decreasing in the number of firms, then increasing the number of firms increases exploitation.

As an aside, one could imagine distributions of the individual taste where there are a small number of customers who have very a strong preference for one firm but most are relatively sensitive to price. In this case, it is possible that increasing the number of firms would increase the equilibrium price, since firms would begin to focus on loyal customers who have the strong preference. This would lead to less exploitation with more firms, but with these parameters, profit margins would also be increasing in the number of firms, and one could argue about whether or not increasing the number of firms is actually an increase in competitiveness.

We generalize our model be supposing that the individual firm taste parameter ϵ_{ik} is distributed according to the distribution function $G : (-\infty, \infty) \to [0, 1]$ for all k, and that ϵ_{ik} is independently distributed for all i and k. Thus for any i and k the likelihood that $\epsilon_{ik} \leq z$ is $G(z)$. We use g to refer to the density of G, so $g(z) = \frac{dG_z}{dz}$. If all other firms are offering a contract with terms p_j and e_j, if firm i offers a contract the profit for firm i is

$$\pi_i = (1-\lambda)(p_i + \alpha f(e_i)) \int_{-\infty}^{\infty} G(\eta + p_j - p_i + \alpha e_j - \alpha e_i))^{J-1} g(\eta) d\eta + \lambda(p_i + f(e_i)) \int_{-\infty}^{\infty} G(\eta + p_j - p_i)^{J-1} g(\eta) d\eta \quad (11)$$

In a symmetric equilibrium where every customer purchases the good the first order condition on price reduces to:

$$\frac{\partial \pi_i}{\partial p_i} = 0 \Leftrightarrow \frac{1}{J} = (p + \beta f(e_i)) \int_{-\infty}^{\infty} g(\eta)(J-1)G(\eta)^{J-2} dg(\eta) \quad (12)$$

The first order condition on exploitation is

$$\frac{\partial \pi_i}{\partial ei} = 0 \Leftrightarrow \frac{\beta f'(e_i)}{J} = \alpha(1-\lambda)(p_i + \alpha f(e_i)) \int_{-\infty}^{\infty} g(\eta)(J-1)G(\eta)^{J-2} dg(\eta) \quad (13)$$

and by solving simultaneously we get the by now familiar expression (10):

$$\beta f'(e_i)(p + \beta f(e_i)) = (1-\lambda)\alpha(p_i + \alpha f(e_i)) \quad (14)$$

Thus except for the fact that the uniform idiosyncratic taste term guarantees that prices will be going down as more firms enter, our result that exploitation tends to increase with competition does not depend on how we specified competition between the firms.

4.2 Unavoidable Exploitation

Our principle results have relied on an assumption that sophisticated customers have some ability to avoid exploitation. However suppliers may find it convenient or necessary to exploit customers in ways that cannot be avoided by sophisticated customers. We might imagine a seller offering a product without a part that is necessary to make it work, or requiring an overpriced subscription be purchased to actually use a product with a low upfront price. It should be pointed out that suppliers would rather use exploitative contract terms that could be at least partially avoided by sophisticated customers. Because sophisticated customers correctly anticipate the costs of being exploited, the seller would rather reduce the degree to which the sophisticated are exploited and increase the price by an equivalent amount.[15]

[15] When a seller wishes to price discriminate against the sophisticated customers, the benefit from the higher effective price to sophisticated customers may outweigh the cost of the distortion. For this reason firms generally have some incentive to prevent sophisticated customers from avoiding the exploitation entirely.

We would model unavoidable exploitation by setting $\alpha = 1$. In a symmetric equilibrium with unavoidable exploitation, if all customers wished to purchase from at least one firm, the first order condition would be given by (10). Solving (10) for the special case where $\alpha = 1$ and $\beta = \lambda + (1-\lambda)\alpha = 1$, we have

$$\beta f'(e_J)(p + \beta f(e_J)) = \alpha(1-\lambda)(p + \alpha f(e_J)) \Leftrightarrow f'(e) = 1 - \lambda \tag{15}$$

In this special case, exploitation is not strictly increasing with competition. Because the firms in a symmetric equilibrium make as much from the sophisticated consumers as they make from the naifs, as the profits per customer decreases, the ratio of how many sophisticates they would give up for one naif stays the same, and exploitation does not increase as firms are added to the symmetric equilibrium.

However, as the number of firms gets large, the symmetric equilibrium breaks down. In a symmetric equilibrium each firm's profit decreases as the square of the number of firms J. However, a firm could get profits of at least $\frac{\lambda}{J} f(\bar{e}) - f(e_J)$ by defecting and using the maximally exploitative contract. Similarly, a firm could obtain a profit of at least $\frac{(1-\lambda)(e_J - f(e_J))}{J}$, by deviating and offering an efficient contract at price $(e_J - f(e_J)$ We note that when $\alpha = 1$ then $f(\bar{e}) < \alpha\bar{e}$, so as J gets large we would have the bifurcated equilibrium detailed in part (c) of Proposition 2.

Whether or not this entails greater welfare loss from exploitation than the oligopoly solution with a symmetric equilibrium depends on the form of the returns from exploitation function $f(e)$. For example, if $f(e)$ is quadratic of the form $f(e) = e - ae^2$, then welfare loss per item purchased will be greater in the limit with many competitors than under the oligopoly with a symmetric equilibrium. In the limit with many competitors the welfare loss is purely from the naive consumers, where it is $\bar{e} - f(\bar{e})$ Since $\bar{e}$ would be $\frac{1}{2a}$, the welfare loss from each naive consumer would be $\frac{1}{4a}$ the average welfare loss would thus be $\frac{\lambda}{4a}$. Under a monopolist who maximized using the interior solution the equlibrium exploitation would solve $2ae = \lambda$, so the welfare loss would be $\frac{\lambda^2}{4a}$. Welfare losses from exploitation are likely to be lower in the bifurcated equilibrium only when there is a kink in the returns to exploitation, so that the contract that a firm would offer to purely naive customers is not much more exploitative than the contract they would offer to a mix of naive and sophisticated customers.[16] In any case, naive customers will face more exploitation under competition than under the oligopoly.

[16] We can show that is of the form $f(e) = e - ae^n$ for any positive n, we can show that aggregate welfare loss from inefficient contracts will be greater when there are many firms than when there is a monopoly $na\bar{e}^{n-1} = 1$, so $\bar{e} = (\frac{1}{na})^{\frac{1}{n-1}}$ Welfare loss from competition is given by $\lambda a(\frac{1}{na})^{\frac{n}{n-1}}$ Turning towards the oligopoly, $e_M = (\frac{\lambda}{na})^{\frac{1}{n-1}}$, and welfare loss from exploitation in the oligopoly is $a(\frac{\lambda}{na})^{\frac{n}{n-1}}$. The ratio of oligopoly to competitive welfare loss is $\lambda^{\frac{1}{n-1}} < 1$

4.3 Partial Naivete

In this section we show that in cases where the 'naive' customers are not entirely naive, and are able to anticipate and avoid some exploitation, our qualitative results still apply. Specifically, we denote the fraction of exploitation naifs anticipate as γ and denote the fraction they are able to avoid by $1 - \alpha_n$. We show that our assumption that naifs are completely unable to observe and avoid exploitation really functions as a normalization.

Proposition 3 *As long as the naifs anticipate a lower fraction of their costs from exploitation($\alpha_n\gamma < \alpha$), and are not able to avoid as much exploitation compared to the sophisticates,($\alpha_n > \alpha$) the assumptions that naifs anticipate no exploitation, and avoid no exploitation is without loss of generality*

Proof:

What we will show is that the partial naivete scenario is equivalent to a full-exploitation scenario with different parameters, which we refer to as pseudo-parameters. Furthermore we show that as long as the original parameters comply with the relevant assumptions, the pseudo parameters will also comply with the necessary assumptions as well.

We use ψ, ϵ and $\phi(\epsilon)$ as pseudo-price, pseudo-exploitation, and returns from pseudo-explopitation function, respectively, we will also use $\aleph$ as pseudo-α .

The pseudo-price ψ is defined by: $\psi = p + \gamma\alpha_n e$. Clearly ψ represents the perceived cost to the naive of a contract with the terms (p, e). We define $\epsilon = \alpha_n(1-\gamma)e$ and $\aleph = \frac{\alpha - \alpha_n\gamma}{\alpha_n(1-\gamma)}$. We know that $0 < \aleph < 1$, because $\alpha_n\gamma < \alpha$, and $\alpha < \alpha_n$

Note that $\psi + \aleph\epsilon = p + \alpha e$, so, $\psi + \aleph\epsilon$ represents the perceived cost to the sophisticated of a contract with terms (p, e). We define our returns to pseudo-exploitation as follows:

$$\phi(\epsilon) = \alpha_n f(e) - \gamma\alpha_n e = \alpha_n\left(f\left(\frac{\epsilon}{\alpha_n(1-\gamma)}\right) - \frac{\gamma}{\alpha_n(1-\gamma)}\epsilon\right) \qquad (16)$$

Note that when the contract offered is (p, e), revenue from a sale to a naif is $\psi + \phi(\epsilon)$, and revenue from a sale to a sophisticate is $\psi + \phi(\epsilon)$. We note that evaluated at $\epsilon = 0$, $\phi'(\epsilon) = \frac{1}{1-\gamma} - \frac{\gamma}{1-\gamma} = 1$. We find $\bar{\epsilon}$ with the equation $\phi'(\bar{\epsilon}) = 0$. Solving we have $f'(e) - \gamma = 0$ So $f'(\frac{\bar{\epsilon}}{\alpha_n(1-\gamma)}) = \gamma$. Note that this implies that if a firm is serving only (partially) naive customers, it would be most profitable for it to offer a contract with *pseudo-exploitativeness* equal to $\bar{\epsilon}$. We also note that $\phi'(\epsilon) = \frac{f'(\frac{\epsilon}{\alpha_n(1-\gamma)})}{1-\gamma} - \frac{\gamma}{(1-\gamma)}$, so $\phi''(\epsilon) = \frac{f''(\frac{\epsilon}{\alpha_n(1-\gamma)})}{\alpha_n(1-\gamma)^2}$, thus $\phi''(\epsilon) < 0$

Thus, we have shown that for any set of partial naivete parameters, in which naifs can anticipate, and avoid some exploitation, described by $\alpha, \alpha_n, \lambda, \gamma$, and $f(e)$, for any contract (p, e), one can construct another set of parameters and contract terms (which we refer to as *pseudo-parameters*) $\aleph, \epsilon, \psi, \phi(\epsilon)$ through an affine transformation. The payoffs for all types with the partial naivete parameters are equivalent to the payoffs in a full naivete scenario where $\alpha = \aleph$, $e = \epsilon$, $p = \psi$, and $f(e) \equiv \phi(\epsilon)$. As long as the partial naivete parameters comply with the necessary conditions, the pseudo-parameters comply with the original assumptions, and all of the results for the full naivete assumption carry through.

4.4 Continuum of Types

We have shown that our results are not dependent on our assumption that naive types are completely insensitive to exploitation. However, our assumption that there are only two-types, although common in the literature, probably does not precisely reflect reality. In this section we consider loosening that assumption to the point where customers can fall at any point on a naivete/sophistication spectrum. We are able to show that price and exploitation equations in a symmetric equilibrium (equations (6) and (10)) have analogues in the continuous type case that can be interpreted the same way and lead to the same comparative statics. Thus our result that exploitation is increasing in competition is robust. Our results regarding bifurcation of the market in proposition (2) are less likely to be robust, since they result in firms offering contracts specifically aimed at one of the two discrete types.[17]

Formally, we consider a continuum of types, indexed by sophistication z, representing the fraction of exploitation that the consumer perceives. A consumer of type z's ability to avoid exploitation is given by $\alpha(z)$ so that when the seller offers a contract pair (p, e) the consumer perceives a cost $p + z\alpha(z)$. We assume that $\alpha(z)$ is decreasing in z, but that $z\alpha(z)$ is increasing in$z$, so that more sophisticated customers always perceive a higher cost of exploitation. We assume that the distribution of z is given by $H(z)$, and the density is $h(z)$ We refer to firm i's market share of type z as $x_i(z)$. If all other firms are charging (p_j, e_j), when $p_i = p_j$, $\frac{dx_i(z)}{dp_i} = -1$

Profit is given by

$$\pi_i = \int_0^1 x_i(z)(p + \alpha(z)f(e))h(z)dz \tag{17}$$

We define $\bar{\alpha} = \int_0^1 \alpha(z)h(z)dz$. Note that $\bar{\alpha}$ is analogous to β in the original, two-type, model, it is the average portion that the customer is exploited.

[17] We might speculate that if there is not much cross-subsidization, if there are a continuum of types, as more and more firms enter, firms might distribute themselves in exploitation space like in a hoteling model. If there is more cross subsidization, firms might just offer maximally exploitative contracts, as they focus on the most profitable, most naive customers

In a symmetric equilibrium $x_i(z) = \frac{1}{J}$ for all z, so profit is just $\pi_i = x_i(z)(p + \bar{\alpha} f(e))h(z)dz$.

Looking at our first order condition in the competitive market, we have the analogue of (6)

$$\frac{d\pi_i}{dp_i} = 0 \Leftrightarrow p + \bar{\alpha} f(e_i) = \frac{1}{J} \tag{18}$$

Now we note that at the symmetric equilibrium, $\frac{dx_i(z)}{de_i} = -\alpha(z)z$. Looking at our first order condition for exploitation.

$$\int_0^1 \alpha(z)z(p + \alpha(z)f(e))h(z)dz = \frac{\bar{\alpha} f'(e)}{J} \tag{19}$$

Let us define $\zeta(z) = \alpha(z)z$, so $\zeta(z)$ is a measure of the perceived cost of exploitation to the consumer.

$\bar{\zeta} = \int_0^1 \alpha(z)zh(z)dz$ So $\bar{\zeta}$ is an analogue of $(1-\lambda)\alpha$, it is the average perceived exploitation. We can write the first order condition for exlpoitation as:

$$f'(e) = \frac{\bar{\zeta}(p + \frac{\int_0^1 \alpha^2(z)zh(z)dz}{\bar{\zeta}} f(e))}{\bar{\alpha}(p + \bar{\alpha} f(e))} \tag{20}$$

We rely on the following lemma:

Lemma 4 $\frac{\int_0^1 \alpha^2(z)zh(z)dz}{\bar{\zeta}} < \bar{\alpha}$

Proof: See appendix.

Since $\frac{\int_0^1 \alpha^2(z)zh(z)dz}{\bar{\zeta}} < \bar{\alpha}$, the right side of (20) is increasing in p, holding $f(e)$ constant, so we have the same comparative statics as we derived from (10). The intuition behind this result is the same as in the two-type case, as competition increases, prices and profits go down, this leads to a greater proportionate decrease in the profitability of the more sophisticated, less profitable customers. Since profits are decreased most from the customers who are most sensitive to exploitation, the firms have less incentive to limit exploitation.

5 Extensions and Conclusion

This paper shows that a presumption that competition will protect naive customers is not only not valid, it is exactly wrong. When customers differ in their vulnerability to exploitation, an oligopolist's incentive to use contract terms to exploit naive customers can be constrained by its desire not to alienate sophisticated customers who can see through the exploitation, and whom it must compensate for the inefficiency. On the other hand, when an industry becomes highly competitive, either producers pursue a symmetric strategy, in which case they will begin losing money on sophisticated customers as more competitors enter, or the market bifurcates, and some firms focus on naive customers. In either case, naive customers will buy exclusively from firms which do not mind alienating sophisticated customers and will be exploited as much as possible.

An alternative route by which competition might be supposed to reduce exploitation is through a firm's incentive to educate consumers about its competitors' exploitative contracts. However, as shown in Gaibson and Labaix[18], in a competitive equilibrium with substantial cross-subsidization firms will actually be losing money on sophisticated customers. If by educating their competitor's naive customers they convert them into sophisticates, they will attract only the wrong kind of customer. By contrast, in a bifurcated equilibrium, a firm that offers the sophisticated contract might have some incentive to educate, however, this incentive is likely to be small. Since the bifurcation tends to occur where there are a large number of firms, there is likely to be a significant free rider problem; any benefits that came from educating the exploitative firms' customers would be spread among all the firms offering the sophisticated contract, not just the firm that invests in education.

The major result in this paper, that competition tends to increase exploitation does not imply a presumption that competition is bad in itself; one would expect that the benefits of lower prices from competition will often outweigh the increased losses from exploitation of the naive, and competition will usually help the sophisticated.[19] Nonetheless this paper provides a strong formal argument that if the courts are concerned with protecting the consumers who have difficulty fully understanding the implication of contract terms, courts should not show any special solicitude to contracts offered by suppliers in competitive industries, nor should courts assume that a firm that provides customers with a choice of contracts is less likely to be exploiting cognitive failures of its customers.

[18] *Supra*, note 7

[19] One could construct parameters where the sophisticated are made worse off by competition. The intuition is that the effective cross subsidization from the naive to the sophisticated (the difference between the cross subsidization and the inefficiency from exploitation) may be maximized at an intermediate level of exploitation. The decrease in effective subsidization from increased exploitation with competition may outweigh the customers increasing share of the surplus.

6 Appendix

6.1 Proof of Proposition 1

We have shown that for any contract pair (p_J, e_J) which maximizes profit for one of J competitors in a symmetric equilibrium, there is a contract pair (p_{J+1}, e_{J+1}) that maximizes profit with $e_{J+1} \geq e_J$ and $p_{J+1} \leq p_J$. However, we must rule out the possibility that there is some other contract pair (p'_{J+1}, e'_{J+1}), that is a symmetric equilibrium for $J+1$ customers where $e_{J+1} < e_J$

We start by solving (10) for p and obtaining

$$p = f(e)\frac{\alpha^2(1-\lambda) - \beta^2 f'(e)}{\beta f'(e) - \alpha(1-\lambda)} \tag{21}$$

We note also that if $p + \beta f(e) > 0$ so that profits are non negative, by (10) $\beta f'(e) < \alpha(1-\lambda)$. Let $\tilde{e}$ be the lowest value of e that satisfies $\beta f'(e) = \alpha(1-\lambda)$ and $v + 1 = f(e)\frac{\alpha^2(1-\lambda)-\beta^2 f'(e)}{\beta f'(e)-\alpha(1-\lambda)}$. Thus $\tilde{e}$ is the lowest level of exploitation that is consistent with a solution to (21) and consistent with some consumers purchasing. For all integer $J \geq 2$ draw a curve in (e,p) space defined by the line $p + \beta f(e) = \frac{1}{J}$. We refer to these as isoprofit curves, because the profits are constant for any symmetric candidate equilibrium on the curve. They slope down and to the right and represent the contracts which satisfy (6) for a given value of J. Since $f(e)$ and $f''(e)$ are continuous we can draw the sideways graph in (p,e) space of $\tilde{p}(e)$ defined by the solution to (21) on the interval $[\tilde{e}, \bar{e}]$. Append a straight line from $(-\alpha f(\bar{e}), \bar{e})$ to $(-\beta f(\bar{e}), \bar{e})$, which represents contract pairs at which the firm would like to increase exploitation, but exploitation is already at its upper bound. Note that $\tilde{p}(e)$ is lower hemicontinuous on $[\tilde{e}, \bar{e}]$ and that at $\tilde{e}$ this curve is above the $J = 2$ isoprofit curve. Furthermore at the point $(-\beta f(\bar{e}), \bar{e})$ it is below the isoprofit curve for any finite J . At any point where the $\tilde{p}(e)$ curve crosses an isoprofit curve for a given J, the first order conditions for price and exploitation are met, and we refer to this as a candidate equilibrium. For a given J we refer to the candidate equilibrium with the highest exploitation as candidate A, and the candidate with the highest price and lowest exploitation as candidate B. There may be other candidates, but we show that no candidate other than A or B can be an equilibrium.

Define the contract pairs associated with candidates A and B as (p_A, e_A) and (p_B, e_B), which both satisfy (8) and (6). We will show that at most 1 can be an equilibrium.

Define $\pi_x(p,e)$ as the profit a producer realizes from offering a contract at (p_x, e_x) while everyone else is offering (p,e). Note that $\pi_x(p_x, e_x) = \frac{1}{J^2}$ We define $\hat{e}(p)$ as the value of e that satisfies (6) for a given p. If $p > p_A$ and $p + \alpha e < p_A + \alpha e_A$, then π_A

is given by

$$\pi_A = (1-\lambda)\frac{(1+p+\alpha e - p_A - \alpha e_A)^J}{J}(p_A+\alpha f(e_A))+\lambda(p-p_A+\frac{1-(p-p_A)^J}{J})(p_A+f(e_A)) \quad (22)$$

Likewise, if $p < p_B$ and $p + \alpha e > p + \alpha e_B$, then π_B is given by

$$\pi_B(p,e) = (1-\lambda)(p_A+\alpha e_A - p_B - \alpha e_B + \frac{1-(p_A+\alpha e_A - p_B - \alpha e_B)^J}{J})(p_B+\alpha f(e_B)) + \lambda\frac{(1+p_A-p_B)^J}{J})(p_B+f(e_B)) \quad (23)$$

Either (Case I) $p_B - p_A > p_A + \alpha e_A - (p_B + \alpha e_B)$, (Case II) $p_B - p_A = p_A + \alpha e_A - (p_B + \alpha e_B)$, or (Case III)$p_B - p_A < p_A + \alpha e_A - (p_B + \alpha e_B)$

Lemma 5 *In Case I only candidate B can be an equilibrium. In Case II there can be no symmetric equilibrium. In Case III, only candidate A can be an equilibrium*

Proof of Lemma:

Suppose we are in Case I or Case II. Consider all pairs (e,p) such that $e = \hat{e}(p)$ and $e \in (e_B, e_A)$. By the convexity of $\hat{e}$, we know $p_B - p > p + \alpha e - p_B - \alpha e_B$. The partials of π_B with respect to price and exploitation are given by:

$$\frac{\partial \pi_B}{\partial p} = (1-\lambda)(1-(p+\alpha e - p_B - \alpha e_B)^{J-1})(p_B+\alpha f(e_B)) + \lambda(1-p_B+p)^{J-1})(p_B+f(e_B)) \quad (24)$$

$$\frac{\partial \pi_B}{\partial e} = \alpha(1-\lambda)(1-(p+\alpha e - p_B - \alpha e_B)^{J-1})(p_B+\alpha f(e_B)) \quad (25)$$

Since $p_B - p > p + \alpha e - p_B - \alpha e_B$, $(1-p_B+p)^{J-1} < 1-(p+\alpha e - p_B - \alpha e_B)^{J-1}$ thus

$$\frac{\partial \pi B}{\partial p} < (1-(p+\alpha e - p_B - \alpha e_B)^{J-1})(p_B+\beta f(e_B)) \quad (26)$$

The change in $\pi_B(p,e)$ as we move along the isoprofit $(\hat{e}(p))$ line towards (p_A, e_A) is given by:

$$\frac{d\pi_B}{dp} = \frac{\partial \pi_B}{\partial p} + \hat{e}'(p)\frac{\partial \pi_B}{\partial e} \quad (27)$$

Note that $\hat{e}'(p) < \hat{e}'(p_B)$. By the definition of $\hat{e}$ as an isoprofit line,$\hat{e}'(p) = \frac{-1}{\beta f'(\hat{e})}$. By (10), at any symmetric equilibrium, $\beta f'(\hat{e}) = \frac{\alpha(1-\lambda)(p+\alpha f(\hat{e}))}{p+\beta f(\hat{e})}$. Thus we have $\hat{e}'(p_B) = \frac{-(p+\beta f(e_B))}{\alpha(1-\lambda)(p+\alpha f(e_B)}$ so

$$\frac{d\pi_B}{dp} <$$
$$(1-(p+\alpha e - p_B - \alpha e_B)^{J-1})(p_B+\beta f(e_B) - \frac{\alpha(1-\lambda)(p_B+\alpha f(e_B))(p_B+\beta f(e_B))}{\alpha(1-\lambda)(p_B+\alpha f(e_B))}) \quad (28)$$

Thus $\frac{d\pi B}{dp} < 0$ for any p on the isoprofit line between p_A and p_B, Since $p_A < p_B$, $\pi_B(p_A, e_A) > \pi_B(p_B, e_B) = \frac{1}{J^2}$. Thus if all competitors are offering contract (p_A, e_A), one could increase profits by deviating to (p_B, e_B), and candidate A cannot be an equilibrium.

Now suppose we are in Case III, again by the convexity of $\hat{e}$ we know $p - p_A < p_A - \alpha e_A - p - \alpha e$ and that $(1 + p + \alpha e - p_A - \alpha e_A)^{J-1} < (1 - (p - p_A)^{J-1})$ .Noting that $\hat{e}'(p) > \hat{e}'(p)$, we can show that $\frac{d\pi_A}{dp} > 0$ on the isoprofit curve from A to B. Thus playing A is a profitable deviation from candidate equilibrium B, and only (p_A, e_A) could be an equilibrium. In Case II both $p - p_A < p_A \alpha e_A - p - \alpha e$ and $p_B - p > p + \alpha e - p_B - \alpha e_B$, thus B is a profitable deviation from A, and A is a profitable deviation from B, and there can be no symmetric equilibrium.

Finally we define J_M as the lowest integer such that $J_M^2 < \frac{1}{(\beta-\alpha)f(\bar{e})}$ and make a continuity argument noting that $(\tilde{p}(\tilde{e}), \tilde{e})$ is above the $J = 1$ isoprofit line and $(-\alpha f(\bar{e}), \bar{e})$ is below the J_M isoprofit line, For any $J \in [2, J_M]$, the pair $(\tilde{p}(\tilde{e}), \tilde{e})$ is above the isoprofit line, and $(-\alpha f(\bar{e}), \bar{e})$ is below the line so the $\tilde{p}$ curve must cross an odd number of times, and both e_A and e_B are increasing in J. We also note that as e_A increases and as e_B increases we move from Case I to Case II and Case III. Define J_1 as the highest J in case I, and J_3 as the lowest value of J that falls in Case III. Compare any J and J' for which there are symmetric equilibria, and without loss of generality, assume $J < J'$. Since there are no equilibria in Case II, either J and J' are both in Case I, J is in Case I and J' is in Case III, or they are both in Case III. If they are both in Case I, the equilibria occur at (p_B^J, e_B^J) and $(p_B^{J'}, e_B^{J'})$ respectively, with $e_B^{J'} > e_B^J$. If J is in Case I and J' is in Case III, the the equilibria occur at (p_B^J, e_B^J) and $(p_A^{J'}, e_A^{J'})$, and $e_B^J < e_A^J < e_A^{J'}$. If they are both in Case III the equilibria occur at the A candidates, and $e_A^J < e_A^{J'}$. Now suppose $J' > J_N$, at the symmetric candidate at $e_A = \bar{e}$, profits are negative from sophisticated customers so they are trivially actually increasing when the competitors choose a more efficient higher priced contract. Thus for any $J\prime > J_M$, a symmetric equilibrium can occur only at e_A as in case III. Thus for any $J < J'$, if there are symmetric equilibria with both J and J' competitors, exploitation is weakly greater with J' competitors, and this inequality is strong if $J < J_M$. QED

6.2 Proof of Proposition 2

[[I apologize for the messiness of this proof, I am in the process of cleaning it up]]

Case (a) If $\beta f(\bar{e}) > \alpha\bar{e}$, at $(-\beta f(\bar{e}) + \frac{1}{J}, \bar{e})$, $\frac{\partial\pi_i}{\partial p_i} = 0$ and $\frac{\partial\pi_i}{\partial e_i} > 0$. As $J \to \infty$ any deviation where $e < \bar{e}$ must charge the same price, or lose all the sophisticated customers. Charging $(-\beta f(\bar{e}) + \frac{1}{J}, e)$ with $e < \bar{e}$ is clearly unprofitable as $J \to \infty$, so any profitable contract with $e < \bar{e}$ must only attract sophisticated customers, and can only attract customers if $p + \alpha e < \alpha\bar{e} - \beta f(\bar{e}) < 0$. But average profit from a sophisticated customer is $p + \alpha f(e) < p + \alpha e$, so no firm can profitably deviate.

Case (b)

Suppose that J is large and that $\beta f(\bar{e}) < \alpha\bar{e} < f(\bar{e})$. For the sake of compactness define $\delta = p_N + \alpha e_N - p_S$, where p_N and p_S are the prices charged in the naive and sophisticated contracts respectively. So δ is the difference in the surplus an average sophisticated customer would get from buying at the average sophisticated contract relative to the average naive contract. Let μ be the fraction of sophisticated consumers who purchase from a firm offering the 'naive' contract, and let $\hat{\mu}$ be the value that satisfies $\frac{\lambda f(\hat{e}) + (1-\lambda)\hat{\mu}\alpha f(\hat{e})}{\lambda + (1-\lambda)\hat{\mu}} = \alpha\hat{e}$

Let us look at the first order conditions. First we compute the sales of a firm (j) offering a naive contract.

$$x_j^N = \lambda \int_0^1 (\zeta + p_N - p_j)^{J_N - 1}) d\zeta \tag{29}$$

$$x_j^S = (1-\lambda) \int_0^{1-\delta} (\zeta + p_N - p_j)^{J_s}) d\zeta \tag{30}$$

We note that $\frac{dx_j^S}{dp_j} = -\lambda$ and $\frac{dx_j^S}{dp_j} = -(1-\lambda)(1-\delta)^{J_S}$, so the first order condition for price for firms offering the 'naive' contract is given by:

$$\frac{\lambda + (1-\lambda)\mu}{J_N} = \lambda(p_N + f(\bar{e})) + (1-\delta)^{J_S}(1-\lambda)(p_N + \alpha f(\bar{e})) \tag{31}$$

If we define ϵ as $p_i - p_S$, then the sales of a firm i offering the sophisticated contract is

$$x_i^S = (1-\lambda)(\int_0^{1-\delta+\epsilon} (\zeta+\epsilon)^{J_N}(\zeta-\epsilon)^{J_S - 1} d\zeta + \int_{1-\delta+\epsilon}^1 (\zeta-\epsilon)^{J_S-1} d\zeta) \tag{32}$$

clearly $\frac{dx}{dp_i} = \frac{dx}{d\epsilon} = -(1-\lambda)$

If μ is the share of sophisticates buying the naive contract we have the first order condition for the firms offering sophisticated contracts:

$$\frac{1-\mu}{J_S} = p_S \tag{33}$$

The total share given to the naive is given by $\mu = \int_\delta^1 J_N \zeta^{J_N - 1}(\zeta - \delta)^{J_S} d\zeta$. Solving the integral, we have $\mu = J_N \sum_{i=0}^{J_S} \frac{J_S!}{(J_S-i)!i!} \frac{(-\delta)^i}{J_S+J_N-i} + \frac{J_S!(J_N-1)!}{(J_S+J_N)!}\delta^{(JS+JN)}$

We note that profits must be similar for both the naive and sophisticated types. By definition $p_N = p_S + \delta - \alpha\bar{e}$, so

$\frac{(1-\mu)(1-\lambda)}{J_S^2} \approx \frac{(\lambda+(1-\lambda)\mu)(\alpha f(\bar{e})-\alpha\bar{e}+\delta+\frac{1-\mu}{J_S})+\lambda(1-\alpha)f(\bar{e})}{J_N}$

We now have four relations we can use to solve for three unknowns

$$J_N + J_S = J \tag{34}$$

$$\mu = \int_\delta^1 J_N \zeta^{J_N-1}(\zeta-\delta)^{J_S} = J_N \sum_{i=0}^{J_S} \frac{J_S!}{(J_S-i)!i!} \frac{(-\delta)^i}{J_S+J_N-i} + \frac{J_S!(J_N-1)!}{(J_S+J_N)!}\delta^{(JS+JN)} \tag{35}$$

$$\frac{(1-\mu)(1-\lambda)}{J_S^2} \approx \frac{(\lambda+(1-\lambda)\mu)(\alpha f(\bar{e})-\alpha\bar{e}+\delta+\frac{1-\mu}{J_S})+\lambda(1-\alpha)f(\bar{e})}{J-J_S} \tag{36}$$

$$\frac{\lambda+(1-\lambda)\mu}{J-J_S} = \lambda(\frac{1-\mu}{J_S}+\delta-\alpha\bar{e}+f(\bar{e})) + (1-\delta)^{J_S}(1-\lambda)(\frac{1-\mu}{J_S}+\delta-\alpha\bar{e}+\alpha f(\bar{e})) \tag{37}$$

Suppose that δ is small, both J's are large but $\frac{J_S}{J_N}$ is also small Suppose further that δJ_S is not large

then $\mu \approx \frac{J_N(1-\delta)^{J_S}}{(J_N+J_S+\delta(1-\delta)J_S\frac{J}{J-1}}$

Since $\delta^2 J_S$ is very small compared to (J_N+J_S) as, is $\frac{J}{J-1}-1$ we can integrate by parts and obtain the approximation

$\mu \approx \frac{J_N(1-\delta)^{J_S+\frac{J_S}{J_N}}}{J}$

We can also write the profits for the naive firm as $\pi_N = \frac{\mu(1-\lambda)+\lambda}{J_N}(\delta+\frac{1-\mu}{J_S}) + \frac{(\hat{\mu}-\mu)(1-\alpha)f(e)(1-\lambda)}{J_N}$

The first order condition can be rewritten by substituting $(1-\delta)^{J_S} = \mu\frac{J}{J_N}(1-\delta)^{\frac{-J_S}{J_N}}$

The first order conditions are now

$$\frac{\lambda+(1-\lambda)\mu}{J_N} = \lambda(\frac{1-\mu}{J_S}+\delta-\alpha\bar{e}+f(\bar{e}))+\mu\frac{J}{J_N}(1-\delta)^{\frac{-J_S}{J_N}}(1-\lambda)(\frac{1-\mu}{J_S}+\delta-\alpha\bar{e}+\alpha f(\bar{e})) \tag{38}$$

Recall that $\lambda(f(\hat{e})-\alpha\hat{e})+(1-\lambda)\hat{\mu}(\alpha f(\hat{e})-\alpha\hat{e}) = 0$

so the FOC becomes

$$\frac{\lambda+(1-\lambda)\mu}{J_N} = (\lambda+(1-\lambda)\mu\frac{J}{J_N}(1-\delta)^{\frac{-J_S}{J_N}})(\frac{1-\mu}{J_S}+\delta)+(\hat{\mu}-\mu\frac{J}{J_N}(1-\delta)^{\frac{-J_S}{J_N}})(1-\lambda)\alpha(\bar{e}-f(\bar{e})) \quad (39)$$

As an aside Clearly as J_N becomes large $\mu\frac{J}{J_N}(1-\delta)^{\frac{-J_S}{J_N}} \to \hat{\mu}$

(note if δ is big , either $\mu \to 0$, in which case almost all firms must be sophisticates, in which case profits for the naive firms would be too big, or J_S is small.)

Suppose J_S remains small as J_N gets big – unless $\mu \to 1$ Total profit is on the order of $\frac{J}{J_S^2}$, so J_S can't stay small – Thus δ also becomes small

Remember that $\pi_N = \frac{\mu(1-\lambda)+\lambda}{J_N}(\delta + \frac{1-\mu}{J_S}) + \frac{(\hat{\mu}-\mu)(\alpha(\bar{e}-f(\bar{e}))(1-\lambda)}{J_N}$

We would really like to get a sense for how

$(\lambda+(1-\lambda)\mu\frac{J}{J_N}(1-\delta)^{\frac{-J_S}{J_N}})(\frac{1-\mu}{J_S}+\delta)$ compares with $(\hat{\mu}-\mu\frac{J}{J_N}(1-\delta)^{\frac{-J_S}{J_N}})(1-\lambda)(1-\alpha)f(\hat{e})$

since $(1-\delta)^{J_S} \approx \mu$ we know that $\delta J_S \approx -\ln\mu$ and that $\delta \approx \frac{-\ln\mu}{J_S}$ so the first order condition becomes something along the lines of

$$\frac{\lambda+(1-\lambda)\mu}{J_N} = (\lambda+(1-\lambda)\mu^{\frac{J_N-1}{J_N}}\frac{J}{J_N})(\frac{1-\mu-\ln\mu}{J_S})+(\hat{\mu}-\mu^{\frac{J_N-1}{J_N}}\frac{J}{J_N})(1-\lambda)\alpha(\bar{e}-f(\bar{e})) \quad (40)$$

Or

$$\lambda+(1-\lambda)\mu = (\frac{\lambda J_N}{J_S}\lambda+(1-\lambda)\mu^{\frac{J_N-1}{J_N}}\frac{J}{J_S})+(J_N\hat{\mu}-\mu^{\frac{J_N-1}{J_N}}J)(1-\lambda)\alpha(\bar{e}-f(\bar{e})) \quad (41)$$

Define z $=1-\mu-\ln(\mu)$ note $z>1$

$$(J_N\hat{\mu}-\mu^{\frac{J_N-1}{J_N}}J) = \frac{\lambda(1-z\frac{J_N}{J_S})+(1-\lambda)\mu(1-z\mu^{\frac{-1}{J_N}}\frac{J}{J_S})}{(1-\lambda)\alpha(\bar{e}-f(\bar{e}))} \quad (42)$$

Recall naive firms profits are roughly given by

$$\pi_N = \frac{\mu(1-\lambda)+\lambda}{J_N}(\delta+\frac{1-\mu}{J_S})+\frac{(\hat{\mu}-\mu)(1-\lambda)\alpha(\bar{e}-f(\bar{e}))}{J_N} \quad (43)$$

Note that by (42)

$$\hat{\mu}-\mu = \frac{\lambda(1-z\frac{J_N}{J_S})+(1-\lambda)\mu(1-z\mu^{\frac{-1}{J_N}}\frac{J}{J_S})}{J_N(1-\lambda)\alpha)(\bar{e}-f(\hat{e}))}+(\mu^{\frac{J_N-1}{J_N}}\frac{J}{J_N}-\mu) \quad (44)$$

so

$$\pi_N = \frac{\mu(1-\lambda)+\lambda}{J_N}\frac{z}{J_S} + \frac{(\lambda(1-z\frac{J_N}{J_S}) + (1-\lambda)\mu(1-z\mu^{\frac{-1}{J_N}}\frac{J}{J_S}))\alpha(\bar{e}-f(\bar{e}))}{J_N^2\alpha(\bar{e}-f(\bar{e}))} + (\mu^{\frac{J_N-1}{J_N}}\frac{J}{J_N}-\mu)\frac{(1-\lambda)\alpha(\bar{e}-f(\bar{e}))}{J_N} \quad (45)$$

$$\pi_N = \frac{\mu(1-\lambda)+\lambda}{J_N}\frac{z}{J_S} + \frac{(\lambda(\frac{J_S}{J_N}-z) + (1-\lambda)\mu(\frac{J_S}{J_N}-z\mu^{\frac{-1}{J_N}}(1+\frac{J_S}{J_N}))}{J_N J_S} + (\mu^{\frac{J_N-1}{J_N}}\frac{J}{J_N}-\mu)\frac{(1-\lambda)\alpha(\bar{e}-f(\bar{e}))}{J_N} \quad (46)$$

Note that $\mu^{\frac{-1}{J_N}} = 1 - \frac{\ln\mu}{J_N}$

Putting it together

$$\pi_N = \frac{z\mu(1-\lambda) + z\lambda + \lambda\frac{J_S}{J_N} - \lambda z + (1-\lambda)\mu(\frac{J_S}{J_N} - z - z\frac{J_S}{J_N} + z\frac{\ln\mu}{J_N} + z\frac{J_S\ln\mu}{J_N^2}}{J_N J_S} + (\mu^{\frac{J_N-1}{J_N}}\frac{J}{J_N}-\mu)\frac{(1-\lambda)(1-\alpha)f(e)}{J_N} \quad (47)$$

Thus we have:

$$\pi_N = \frac{\lambda\frac{J_S}{J_N} + \mu(1-\lambda)(\frac{J_S}{J_N} - z(\frac{J_S-\ln(\mu)}{J_N}))}{J_N J_S} + (\mu^{\frac{J_N-1}{J_N}}\frac{J}{J_N}-\mu)\frac{(1-\lambda)\alpha(\bar{e}-f(\bar{e}))}{J_N} \quad (48)$$

Replacing $\mu^{\frac{J_N-1}{J_N}}\frac{J}{J_N}$ with $\mu(1-\frac{\ln\mu}{J_N}+\frac{J_S}{J_N})$, we have:

$$\pi_N = \frac{\lambda\frac{J_S}{J_N} + \mu(1-\lambda)(\frac{J_S}{J_N} - z(\frac{J_S+\ln(\mu)}{J_N}))}{J_N J_S} + \mu\frac{J_S-\ln\mu}{J_N}\frac{(1-\lambda)\alpha(\bar{e}-f(\bar{e}))}{J_N} \quad (49)$$

As J_N and J_S become big this is approximately

$$J_N^2\pi_N = \lambda + \mu(1-\lambda)(1-z) + \mu(1-\lambda)\alpha(\bar{e}-f(\bar{e}))(J_S-\ln\mu) \quad (50)$$

Recall that $\pi_N \approx \pi_S \approx (1-\lambda)\frac{(1-\mu)^2}{J_S^2}$

so

$$\frac{(1-\mu)^2(1-\lambda)}{J_S^2} \approx \frac{\lambda+\mu(1-\lambda)-z+\mu(1-\lambda)\alpha(\bar{e}-f(\bar{e}))(J_S-\ln\mu)}{J_N^2} \quad (51)$$

First Order Approximation is

$$\frac{(1-\lambda)(1-\mu)^2}{J_S^2} \approx \mu \frac{J_S}{J_N} \frac{(1-\lambda)\alpha(\bar{e}-f(\bar{e}))}{J_N} \tag{52}$$

so

$$J_S^3 \approx \frac{(1-\mu)^2 J_N^2}{\mu\alpha(\bar{e}-f(\bar{e}))} \tag{53}$$

This Implies that $J_S^3 \sim J_N^2$

(c) If $f(\bar{e}) < \alpha\bar{e}$ then a firm that is offering naive contracts to only naive customers at the break even price would not be able to draw any sophisticated customers from firms offering the sophisticated contract at $p_S \approx \frac{1}{J_S}$.

Consider the equilibrium where $p_N = \frac{1}{J_N} - f(\bar{e})$ and $p_S \approx \frac{1}{J_S}$, where $\frac{J_N^2}{J_S^2} = \frac{\lambda}{1-\lambda}$

The fraction of naive customers choosing the naive contract is at least $1-(1-f(\bar{e}))^{J_N}$, clearly this approaches 1 as J_N gets large . Likewise the fraction of sophisticated customers choosing the sophisticated contract is at least $1-(1+f(\bar{e})-\alpha\bar{e})^{J_S}$, Since $(1+f(\bar{e})-\alpha\bar{e}) < 1)$ by condition (c), this approaches 1 as J_S becomes large. For the sophisticated firms the price which satisfies the first order condition is bracketed b:.

$$\frac{1}{J_S} > p_S > \frac{1-(1+f(\bar{e})-\alpha\bar{e})^{J_S}}{J_S} \tag{54}$$

For naive firms, the equivalent bracket is given by:

$$\frac{1}{J_N} > p_N > \frac{1-(1-f(\bar{e}))^{J_N}}{J_N} \tag{55}$$

The market shares for a sophisticated and naive firm respectively are given by $x_N \approx \frac{\lambda}{J_N}$ and $x_S \approx \frac{1-\lambda}{J_S}$. Profits are thus approximately given by

$$\pi_N \approx \frac{\lambda}{J_N^2} \tag{56}$$

and

$$\pi_S \approx \frac{1-\lambda}{J_S^2} \tag{57}$$

It is easy to see that $\pi_N \approx \pi_S \Rightarrow \frac{J_N^2}{J_S^2} \approx \frac{\lambda}{1-\lambda}$

6.3 Proof of Lemma 4

We must compare $\bar{\alpha}\bar{\zeta}$ to $\int_0^1 \alpha^2(z)zh(z)dz$. We rewrite $\int_0^1 \alpha^2(z)zh(z)dz$ as $\int_0^1 \alpha(z)\zeta(z)h(z)dz$

Define $z^\dagger$ so that $\alpha(z^\dagger) = \bar{\alpha}$. By the definition of $\bar{\zeta}$

$$\bar{\alpha}\bar{\zeta} - \int_0^1 \alpha^2(z)z)h(z)dz = \int_0^1 (\bar{\alpha} - \alpha(z))\zeta(z)h(z)dz \tag{58}$$

This can be written

$$\int_{z^\dagger}^1 (\bar{\alpha} - \alpha(z))\zeta(z)h(z)dz - \int_0^{z^\dagger} (\alpha(z) - \bar{\alpha})\zeta(z)h(z)dz \tag{59}$$

We note that by the definition of $\bar{\alpha}$

$$\int_{z^\dagger}^1 (\bar{\alpha} - \alpha(z))h(z)dz - \int_0^{z^\dagger} (\alpha(z) - \bar{\alpha})h(z)dz = 0 \tag{60}$$

so

$$\int_{z^\dagger}^1 (\bar{\alpha} - \alpha(z))\zeta(z^\dagger)h(z)dz - \int_0^{z^\dagger} (\alpha(z) - \bar{\alpha})\zeta(z^\dagger)h(z)dz = 0 \tag{61}$$

However

$$\int_{z^\dagger}^1 (\bar{\alpha} - \alpha(z))\zeta(z)h(z)dz > \int_{z^\dagger}^1 (\bar{\alpha} - \alpha(z))\zeta(z^\dagger)h(z)dz \tag{62}$$

and

$$\int_0^{z^\dagger} (\alpha(z) - \bar{\alpha})\zeta(z)h(z)dz < \int_0^{z^\dagger} (\alpha(z) - \bar{\alpha})\zeta(z^\dagger)h(z)dz \tag{63}$$

So

$$\bar{\alpha}\bar{\zeta} - \int_0^1 \alpha^2(z)z)h(z)dz = \int_{z^\dagger}^1 (\bar{\alpha} - \alpha(z))\zeta(z)h(z)dz - \int_0^{z^\dagger} (\alpha(z) - \bar{\alpha})\zeta(z)h(z)dz > 0 \tag{64}$$

Q.E.D.